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Asset Allocation for Retirement:
a utility approach

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Abstract

This thesis examines how different asset allocation strategies impact the terminal wealth of individuals at retirement. We are particularly interested in studying the benefits and disadvantages of age-variant strategies, as their use has been growing. To do this, we analyze how several strategies have performed over historical rolling periods (1928 to 2013), and using a simulation we predict how they will fare in the future. The results suggest that at modest levels of risk aversion fix asset allocation strategies dominate. However, for highly risk-averse individuals, age-variant strategies may improve terminal wealth up to 3.1% when compared with the next best age-invariant strategy.

Keywords: Asset Allocation, Glidepath, Isoelastic Utility, Retirement

1. Introduction

“However beautiful the strategy you should occasionally look at the results.”

- Winston Churchill

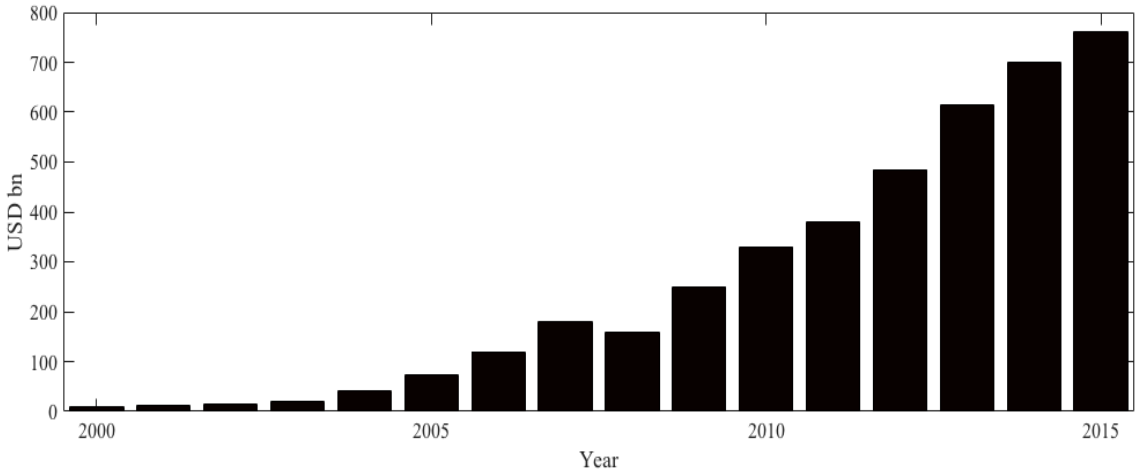
Saving for retirement is not enough. Planning where to apply savings is a decision as important. If savings are not invested over a long period, investors lose purchasing power due to inflation. The alternative is to allocate saving on assets that provide a return, but that usually comes with added risk of losing part, or all, of the invested amounts. Thus, getting this allocation right is vital to attaining financial security after retirement.

The fact that different strategies result in very different retirement wealth outcomes has prompted great interest in determining the optimal allocation of wealth for retirement purposes. An intuitive and widely used approach is to advise investors to allocate their wealth heavily on high-return and risky assets - namely on the stock market - at the beginning of their lives and gradually shift to safer - and lower yield - assets as retirement approaches. This strategy is most commonly known as Glidepath. Other options may consist in allocating fixed weights over time to the assets, with or without rebalancing.

A new and growing offer in the financial services market are the “life-cycle funds” that automatically change the asset allocation across time, basically automatizing the weight allocated to different asset classes depending on the investor’s age or time left until retirement. Most of these funds are thus offering a version of the Glidepath Strategy, without the hassle of taking periodic active decisions about asset allocation. As shown in

Figure 1, these funds have consistently been growing, reaching approximately 763 billion USD[1] of assets under management in 2015.

Figure 1: Life-cycle funds' Assets under Management



The goal of this thesis is to study the effect of different saving allocation under these strategies on the distribution of final wealth. That is, we aim at examining the Glidepath Strategy, and other asset allocation strategies using historical returns, both by hypothetically analyzing the outcomes that they would have generated in the past, and simulating their outcomes in the future if the distribution of future returns is maintained. In the end, we judge the effectiveness of strategies based on the Glidepath, such as the ones used by "life-cycle funds", and evaluate their advantages and disadvantages comparatively to fixed asset allocation.

The results of this work can be used by investors to improve the allocation of their retirement assets, as well as by financial institutions to develop products that mimic these desired strategies or fine-tune the ones they already offer. These institutions may also use it to better assess and communicate the risks and benefits associated with these plans.

[1] Source: Morningstar data.

2. Literature Review

As already stated, age-variant allocation strategies are on the rise, but their benefits haven't yet been completely studied. Arnott (2012) argues, using historical returns on US companies and bonds over 141 years, that the Glidepath Strategy is inferior to its opposite strategy – decreasing stock allocation with time - or even a 50/50 static portfolio. The researcher uses a bootstrapping approach with 101 overlapping return series – or prototype workers – each with 41 consecutive years, starting in 1871 and ending in 2011. He then compares what would have been the terminal wealth at the moment of retirement using a Glidepath and an Inverse-Glidepath, as well as a 50/50 balanced portfolio strategy. The author finds evidence that the Glidepath Strategy is inferior, as it not only produces the lowest average but also the lowest 10th percentile and minimum values of terminal retirement assets when compared with the alternatives.

Estrada (2013) finds that Arnott's (2012) statements about the Glidepath Strategy are true not only when using US historical returns but also in a comprehensive sample of 18 other countries. Emulating Arnott (2012) with data from 1900 to 2009, he argues that giving higher weight to stocks over bonds provides investors with not only higher expected terminal wealth and higher upside potential but also lower downside potential. Here uncertainty comes mostly from upside gains.

Dolvin, Templeton and Rieber (2010) also use the bootstrapping approach, coupling it with a Monte Carlo Simulation to assure the strength of their results. Differently from previous research, they use the ratio of mean over the standard deviation of the terminal value to rank the different strategies analyzed. This method to rank the alternative portfolio allocations results in the same order of preference as ranking average returns, which is considered one of the main drawbacks of this approach. They find that age-dependent strategies are equivalent to static strategies in terms of their final outcomes.

Finally, Poterba et al. (2006) create a more comprehensive approach. They use real historical wage income processes to perform their simulations and make use of isoelastic[2] utility functions to rank several strategies. They analyze extreme asset allocation strategies, investing all in stocks or all in bonds, and several "life-cycle" fund strategies. Ultimately, they find that, at low levels of risk aversion, the all equity plan is superior. But if the historical returns on stocks are reduced by 300 basis points, other strategies become superior to the all equity one.

3. Methodology

To analyze the issue of optimal asset allocation we employ two approaches. First, we examine how each strategy has fared over historical periods, using actual annual real returns from 1928 to 2013. Secondly, we extend this analysis by applying a simulation method that uses the historical distribution of returns as well as a revised forecast for returns.

3.1 Historical Analysis

Using a similar procedure to Arnott (2012), we create 46 consecutive and overlapping "representative workers". The first one starts to save in 1928 and enters retirement in 1968. The second worker saves from 1929 to 1969, and the last one from

[2] Meaning that the decision-making is unaffected by scale, i.e., optimal asset allocation is independent of the initial wealth level.

1973 to 2013. This approach will allow us to analyze how different strategies would have fared in the past.

For each "representative worker", thirteen asset allocation strategies are applied. The first strategy allocates nothing to stocks, the second allocates 10%, the third 20%, up until the eleventh strategy which applies 100% of the savings on stocks. The remainder of the wealth of the individual is applied on bonds and these shares remain fixed through the life of the individual, with rebalancing of the portfolio happening at the beginning of each year. The twelfth strategy is the Glidepath Strategy. In the first year, the individual allocates 20% of his or her assets to bonds and the remaining to stocks. In each year from then onwards, he or she increases the share of bonds by 1.5%, as such, allocating 80% to bonds in the last investment period. The thirteenth strategy is the symmetrical of the Glidepath, starting with an 80% allocation to bonds and decreasing it over time. This approach is called the Inverse Glidepath[3].

Each saver is assumed to save $1,000 at the beginning of each year in real terms. Which is then applied according to the strategy followed. For instance, if the investor follows the Inverse Glidepath, in the first year $200 (20 percent) will be allocated to bonds and $800 (80 percent) to stocks. At the beginning of the next year, the saver will add $1000 more to his savings and rebalance his portfolio. This behavior will be repeated each year for a total of 40 consecutive years. The agent will retire at the end of the 40th year, having contributed with a total of $40,000. In the end, we analyze the final wealth

[3] Other strategies similar to the Glidepath and Inverse Glidepath were also analyzed (with both stronger and weaker equity allocation), but they did not offer addition knowledge and were as such left out of this study.

that each individual would have attained if he had followed a given strategy. A saver's Wealth at the beginning of year *t* is, therefore:

$$W_i(t) = W_i(t-1)\times[1 + R_i(t-1)] + C_i \ , \tag{1}$$

where C_i is his annual savings directed towards retirement and R_i is the return of the portfolio. The return of the portfolio is defined as:

$$R_i(t) = \sum_{n=1}^{N} w_n \times r_n \ , \tag{2}$$

where w_n is the weight given to asset class n and r_n the return that that asset class yields.

With this information, we analyze the distribution of outcomes that each strategy generates. We then calculate the utility of terminal wealth (*W(T)*) in each path and strategy. The utility is described by a constant relative risk aversion function:

$$U\{W(T)\} = \begin{cases} \dfrac{W_i(T)^{1-a} - 1}{1-a} & for\ a \neq 1 \\ \ln\big(W(T)\big) & for\ a = 1, \end{cases} \tag{3}$$

where α is the constant relative risk aversion (CRRA) coefficient. The expected utility is evaluated for of each portfolio strategy as the probability-weighted average utility associated with that strategy. This process is repeated for different values of relative risk aversion. The interval of values used varies from 0 – a risk-averse individual - to as high as 8. Financial literature, such as Chiappori and Paiella (2006) seems to confirm that households' risk aversion – when excluding certain types of assets such as human capital – fall under a constant relative risk aversion[4], and that the risk coefficient varies from

[4] See Chiappori and Paiella (2006).

individual to individual. Chiappori argues that the median alfa sits below 2 and that individuals with CRRA higher than 8 are extremely rare.

Finally, to evaluate these portfolios, we compute the equivalent certain wealth that would generate the same utility level as the terminal wealth distribution attained by that portfolio allocation and stochastic returns. The equivalent certain wealth, Q, is computed as follows:

$$Q\{\,E(U)\} = [E(U)\times(1-a)+1]^{\frac{1}{1-a}} \quad . \tag{4}$$

For instance, if we consider a risk neutral investor (with α=0) following a strategy that pays \$100 or \$200 with the same probability, the equivalent certain wealth, Q, is equal to \$150. On the other hand, if the relative risk aversion coefficient, α, increases to 4 (i.e., the investor becomes more averse to risk), the certainty value falls to \$121,14. (See Table 2 for other values of risk aversion)

After evaluating all portfolios for each type of investor, we rank all strategies according to their equivalent certainty value.

3.2 Monte Carlo Simulation

The simulation method is used to ensure the strength of the results we found in the historical analysis, and to provide more detailed information about each strategy extreme outcomes that could have happened under the same returns distribution. A few downsides can be attributed to using this method: 1) we will lose the characteristics of the historical data that we fail to incorporate in our assumptions[5]; and 2) the results of the Monte Carlo will only be as good as the return's assumptions. Despite that, we are

[5] We only incorporate the average return, standard deviation of each asset class and the cross-correlation between asset classes.

confident that the advantages of such approach justify its use. One of the main reasons for using the simulated method is the fact that it opens the possibility to a more diverse spectrum of scenarios, not limited by the historical returns – which only produce 47 paths with 40 years each. Furthermore, the historical returns limit us to 47 paths which are overlapped, giving a disproportional importance to the middle section of these historical records. For instance, all years from 1968 to 1975 influence the final wealth for 40 individuals while the years of 1928 and 2013 only impact one. Thus, contrary to the historical returns approach, the Monte Carlo simulation treats each historical return that has occurred equally, i.e. giving it the same weight as any other.

The first Monte Carlo simulation is built using the mean, standard deviation, and cross-correlation[6] of the historical returns between stocks and bonds, which are presented in Tables 2 and 3. This simulation[7] produces one million paths, each with 40 years of returns for both stocks and bonds. After the pseudorandom variables generation, we apply the same strategies as used in the historical analysis. We also compute terminal wealth and utility using the same method.

3.3. Data

Historical returns for stocks and bonds were collected from Stern NYU professor Aswath Damodaran' database[8]. We use the S&P500 as a proxy for stock returns and the 10-year US T-Bonds as a proxy for bonds' returns. In total, this corresponds to 86 years of returns, from 1928 to 2013. This data is expressed with annual frequency. We also use

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[6] Autocorrelation features were not included, as both data series failed to reject the non-existence of autocorrelation.
[7] Using MatLab software.
[8] Which has available returns from the S&P500 and 10-Year US T-Bonds having similar returns' distribution to paid sources like Ibbotson Associates, the one used by all Arnott (2012), Estrada (2013) and Dolvin et al. (2010).

the All Urban Consumers CPI, which is collected from the US Bureau of Labor Statistics. This data is used to convert nominal returns from stocks and bonds to real returns. During this period, the stock real returns' distribution is characterized, as expected, by having a higher average return and standard deviation than real bond returns, as it can be seen on Tables 3 and 4.

Looking at the past to predict the future always has its problems. Still, imperfect historical data remains the only unbiased way to measure risk and make assumptions about the future. We use it to create our base model, but due to its shortcomings, a new scenario with a lower return on stocks and bonds is also simulated. In this new scenario, the real return on stocks is decreased to 4.5% per year, and the real return on bonds to 0.5%. The value for stocks returns is proposed by Laurence Siegel. The researcher analyses investors' perceptions (4.7%), dividend and earnings discount models (around 4%) and the last 10 years S&P500 returns (4.6%). He predicts that stocks' real returns will range between 4% and 5% in moderate projections and between 5.5% and 6.5% in the optimist scenario. We use the scenario with the lowest values as a stress test for or predictions. The predictor for the real returns on bonds is the current return on the 10-Year Inflation Protected Securities. The standard deviation and the cross-correlation between stocks and bonds are maintained from the historical period.

4. Results

4.1 Historical Analysis

4.1.1 Distribution of Results

We start by analyzing the performance of each strategy over the historical period studied. Regarding mean terminal wealth (Table 4), the strategy with the best performance is the one that allocates 100% of savings into stocks, with workers ending on average with $207,453. As it would be expected, strategies with lower shares of stocks yield lower average terminal wealth. For instance, if all savings were invested in bonds, the average terminal wealth attained would be $62,734. We can also see that, as expected, the dispersion of average terminal wealth also increases as more wealth is allocated to stocks. We can also observe that only individuals that allocated less than 40% to stocks had, in the worst scenario, ended with lower terminal wealth in real terms than the amount they have saved.

By looking at the individual performance we can also see that the strategy that allocates 100% to bonds always yielded the worst result, and the strategy that assigns all wealth to stocks always yielded the best result except for the last "representative worker" analyzed (retiring in 2013), here the Glidepath Strategy becomes superior, as it protected the worker from the stock declines of 2000-02 and 2008.

4.1.2 Utility

When looking at the utility and equivalent certain wealth (Table 4) attained by each strategy, the strategy that allocates all wealth to stocks is superior to other strategies for all values of risk aversion studied. Even for an individual with a coefficient of risk aversion, α, equal to 8 (the highest value studied), this tactic produces an equivalent certain wealth 10.4% higher than the next best strategy (allocating 90% to stocks and 10% to bonds).

This apparent preference for stocks lies on the structure of the historical returns. As already discussed, the use of rolling windows greatly favors the middle section of the returns distribution, while neglecting the extremes. This strongly skews the results, as the periods from 1942 to 1965 and from 1975 to 1999, were characterized by extremely bullish stock markets returns – both at the center of the period analyzed – while 1929 to 1941, 2000 to 2002 and 2008 to 2011 where bad periods for stocks – both at the "wings" of the historical series. On top of this, the best period for holding bonds was from 1929 to 1940 and from 1984 to 2002, both at the extremes of the sample.

4.2 Simulation

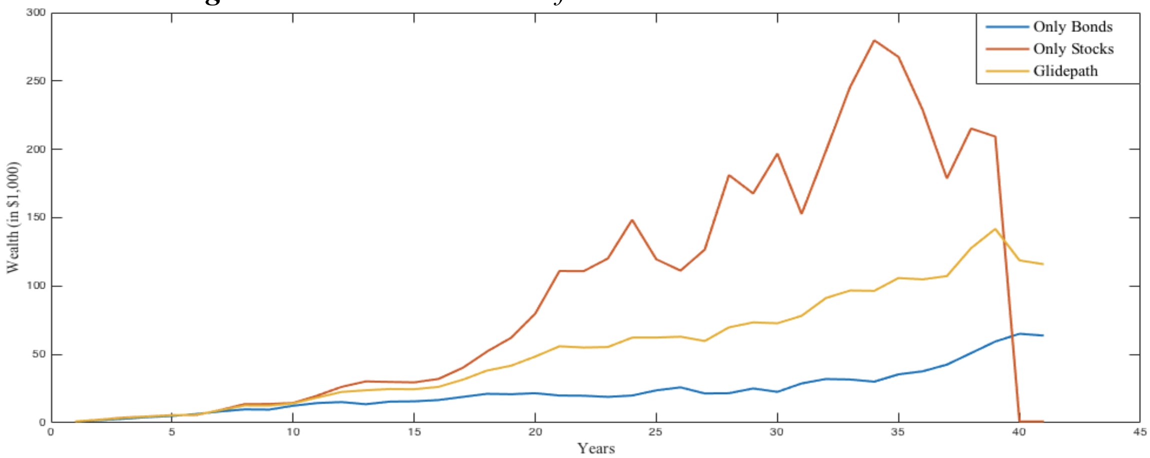
4.2.1 Distribution of Results

Table 5 provides the main results from the simulated series. As in the historical analysis, the strategy that allocates all wealth to stocks still provides the highest mean, \$312,415, considerably more than the average \$63,817 when all savings are allocated to bonds, or the \$123,756 and \$149,996 when using, respectively, the Glidepath and the Inverse Glidepath strategy. Regarding the highest outcome, applying every single penny to stocks resulted, with a one in a million chance, into a maximum terminal wealth of \$31,257,899, which corresponds to more than 781 times the saved amount of \$40,000. Regardless of this, we should also note that this strategy also produced the lowest terminal wealth, again a one in a million chance of ending with, just \$963, which is almost 42 times less than the amount saved. The strategy that results in the highest minimum is the one that applies 80% to bonds and 20% to stocks, which results in a terminal wealth of \$18,388, clearly demonstrating the benefits of diversification.

We further analyze the extreme results produced by the simulation. As seen before, in the worst scenario, when applying 100% of saving to stocks, the individual ended retirement with only \$963. On the other hand, if he had chosen the 100% bond allocation – under the same scenario returns - he would have retired with \$63,999.

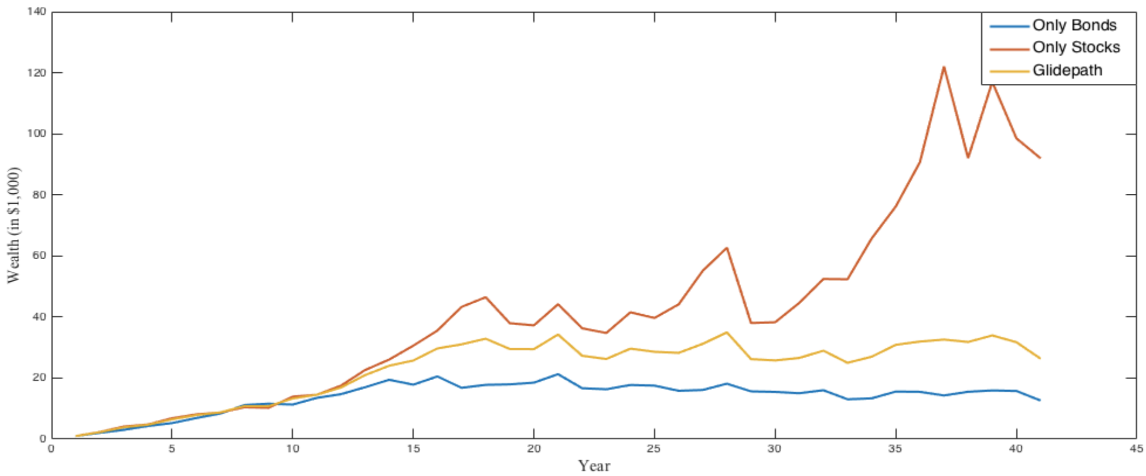
Furthermore, by choosing the Glidepath strategy, the equivalent certain wealth would be of $115,976. These three different paths are illustrated in Figure C.

Figure C *- Wealth over Time for the worst 100% stockholder outcome*



A review of the results provides some meaningful insights to the benefits of the Glidepath Strategy, as it clearly protects the worker against strong falls in the stock market, especially later in life. On top of that, the strategy also generates higher terminal wealth than the all bonds strategy. We can also see this effect when we examine the worst scenario when applying all savings in bonds (plotted in Figure D). In this case, the individual retires with only $12,644. Had he chosen the Glidepath Strategy, he would have retired with $26,352 instead.

Figure D *- Wealth over Time for the worst 100% bondholder outcome*



4.2.2 Utility

Regarding the utility and the equivalent terminal wealth (Table 5 and Figure A), for risk aversion coefficients between 0 and 2.1 – individuals with no risk aversion or low levels of it - the all stocks strategy seems to be the best choice. After this, the optimal stock share the portfolio starts to fall. For instance, for an alfa of 4, a 60/40 stock and bond allocation yields the best results. The most interesting results are related to alphas higher or equal to 5.3. In these cases, the Glidepath comes as the superior strategy. For an individual with a coefficient of risk aversion equal to 8 the equivalent certain wealth created by this portfolio is 3.1% higher than the second-best strategy ($66,969 vs. $64,941 for the 40% stocks and 60% bonds portfolio). These results, when combined with evidence from previous research that individuals' CRRA rarely exceeds 5, supports the idea that the Glidepath Strategy does not suit most individuals' needs. Albeit, it must be noted that for certain individuals – highly risk-averse one – this might be their best investment option, just not the average Joe's solution.

4.3 Simulation with reduced returns

When we perform our simulation with the lower returns distribution, the results (Table 6 and Figure B) skew in favor of bonds. This is likely due to the fact that their returns did not decrease as much as for stocks. Still, for individuals with CRRA below 1.5 the optimal strategy remains the all-stocks portfolio. From there on, the allocation to bonds starts to increase, reaching a 50/50 allocation for risk aversion between 3.1 and 3.4. For higher values of alpha, the optimal choice becomes the Glidepath.

5. Conclusion

This thesis presents the distribution of wealth associated with retirement savings. Various asset allocation strategies were considered, from static or age-invariant to “life-cycle” ones that automatically change the investor’s exposition according to their age.

We found that, when using a utility approach to rank asset allocation strategies, the Glidepath is found to be superior for some individuals. For risk-neutral investors, age-variant strategies are not optimal, as they reduce the exposure to equity and, as consequence, the average terminal wealth. This statement seems to be true also for individuals with low levels of risk aversion. They could benefit from some risk diversification by incorporating fixed income instruments into their portfolio, but the cost of decreasing their equity allocation – and as result the average return - is too high for this trade to happen. Even when exposure to fixed income products becomes beneficial, for individuals with moderate levels of risk aversion, age-invariant asset allocation strategies dominate. It is only when analyzing highly risk-averse agents – with CRRA higher than 5.2 in the base scenario, or higher than 3.4 in the reduced returns scenario - that age-variant strategies - in particular, the Glidepath - outperform the others.

It appears, therefore, that ruling the Glidepath strategy out of the bundle of offering to savers makes them worse off. In fact, we find that for highly risk-averse individuals (CRRA=8) this strategy generates an equivalent certain wealth 3.1% higher than the next best alternative with age-invariant allocation.

Expected utility generated from these investments is sensitive to two main variables: the risk aversion of the investor and the expected future returns. Increasing risk aversion has the consequence of decreasing equity allocation. Assuming historical returns, increases in risk aversion shift the optimal strategy from an all equity exposure, to the inclusion of bonds, and ultimately the adoption of the Glidepath strategy. As such, results suggest that age-invariant strategies may produce added value to highly risk-averse investors. They function as a "gamble" when the individual is young and has few savings, and act as a safe haven when the amount saved is higher. Decreasing these assets' future returns – with a higher decrease in stocks – leads to a quicker transition from all equity to mixed strategies when increasing the risk aversion. Consequently, the Glidepath strategy becomes more attractive, for a larger number of individuals.

Our study also points to relevant future issues that should be studied. One of them is the existence of fees and other expenses associated with asset management, which we do not cover. These costs can take the form of a percentage of the return, the invested amount or fixed value. Consequently, they can change the incentives to follow one strategy vs. another. Another important one is the savings period length, we assumed constant contributions during 40 years, but investors follow a much more diverse contribution path. This can be either due to an income and saving process that is not constant, and with a different length, most usually shorter. Finally, a third and important area of study should be the correlation between the asset classes. We assumed it as constant when, historically, it has varied across time. A relevant continuation of this work would be to simulate correlation paths and incorporate those into the simulated results.

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Table 1
Equivalent Reductions in Wealth

	50% probability of losing			10% probability of losing		
	10%	30%	50%	10%	30%	50%
Risk Aversion Coefficient						
0	5.00%	15.00%	25.00%	1.00%	3.00%	5.00%
1	5.13%	16.33%	29.29%	1.05%	3.50%	6.70%
2	5.26%	17.65%	33.33%	1.10%	4.11%	9.09%
4	5.52%	20.06%	39.43%	1.21%	5.67%	16.21%
8	6.00%	23.58%	44.86%	1.48%	10.18%	31.20%

This table illustrates the equivalent certain wealth reduction that individuals are willing to pay to avoid a certain percentage loss in their wealth, with a given probability, for different levels of CRRA. For a CRRA of 0, this corresponds to the expected loss.

Table 2
Returns' Descriptive Statistics

	Arithmetic Mean	Standard Deviation	Geometric Mean	Min	Max
Nominal Variables					
Stocks	11,50%	20,02%	9,55%	-43,84%	52,56%
Bonds	5,21%	7,85%	4,93%	-11,12%	32,81%
Inflation	3,16%	4,16%	3,07%	-10,06%	18,13%
Real Variables					
Stocks	8,35%	20,31%	6,33%	-37,70%	53,31%
Bonds	2,06%	9,10%	1,66%	-15,00%	29,10%

Table 3
Correlation Matrices

Nominal Returns			
	Stocks	Bonds	Inflation
Stocks	1		
Bonds	-0,0298	1	
Inflation	0,0334	-0,0606	1
Real Returns			
	Stocks	Bonds	
Stocks	1		
Bonds	0,0641	1	

Table 4

Historical Analysis - Descriptive Statistics of Terminal Wealth and Equivalent Terminal Wealth

	Terminal Wealth Distribution					Equivalent Terminal Wealth depending on α			
Share of Equity	Mean	St Dev	Min	Median	Max	0	1	4	8
0	62,734	28,733	23,770	57,172	114,541	62,734	56,134	41,698	34,469
10	70,975	31,073	28,357	63,927	122,374	70,975	64,144	48,670	40,394
20	80,280	33,247	33,219	71,518	134,394	80,280	73,342	56,915	47,447
30	90,764	35,209	39,092	80,023	146,395	90,764	83,842	66,597	55,828
40	102,549	36,973	46,170	94,838	158,084	102,549	95,740	77,861	65,748
50	115,760	38,672	54,668	109,294	179,016	115,760	109,112	90,813	77,405
60	130,524	40,646	64,819	126,844	207,871	130,524	124,002	105,488	91,022
70	146,966	43,522	76,864	151,076	240,279	146,966	140,411	121,813	107,025
80	165,204	48,241	91,048	159,561	276,441	165,204	158,290	139,556	125,864
90	185,339	55,897	107,608	175,070	316,522	185,339	177,526	158,266	147,252
100	207,453	67,473	126,752	181,612	361,954	207,453	197,931	177,206	162,580
Glidepath	110,864	34,475	56,394	100,928	177,234	110,864	105,616	91,595	79,907
Inv Glidepath	119,896	46,310	52,495	115,700	230,212	119,896	110,883	87,496	73,107

(In thousands of dollars)

Table 5

Simulation with Historical Returns - Descriptive Statistics of Terminal Wealth and Equivalent Terminal Wealth

	Terminal Wealth Distribution					Equivalent Terminal Wealth depending on α			
Share of Equity	Mean	St Dev	Min	Median	Max	0	1	4	8
0	63,817	24,369	12,644	62,383	494,021	63,817	59,727	49,494	40,636
10	73,736	26,887	15,195	71,102	445,501	73,736	69,369	58,254	50,385
20	85,517	31,728	18,388	83,656	472,892	85,517	80,280	66,980	59,562
30	99,534	40,063	17,275	97,414	627,048	99,534	92,480	74,959	64,941
40	116,237	53,179	15,757	113,824	930,033	116,237	105,941	81,435	64,685
50	136,168	72,613	14,337	133,425	1,649,507	136,168	120,580	85,738	60,174
60	159,980	100,434	11,455	156,866	3,066,850	159,980	136,253	87,407	53,421
70	188,462	139,579	8,302	184,932	5,615,638	188,462	152,744	86,280	45,372
80	222,562	194,252	5,895	218,570	10,117,858	222,562	169,767	82,500	36,353
90	263,421	270,475	4,085	258,920	17,931,304	263,421	186,958	76,441	27,087
100	312,415	376,876	963	307,357	31,257,899	312,415	203,882	61,474	6,927
Glidepath	123,756	61,863	15,730	121,146	1,284,819	123,756	111,299	83,577	66,969
Inv Glidepath	149,996	93,167	7,692	147,118	2,737,687	149,996	127,615	79,720	43,815

(In thousands of dollars)

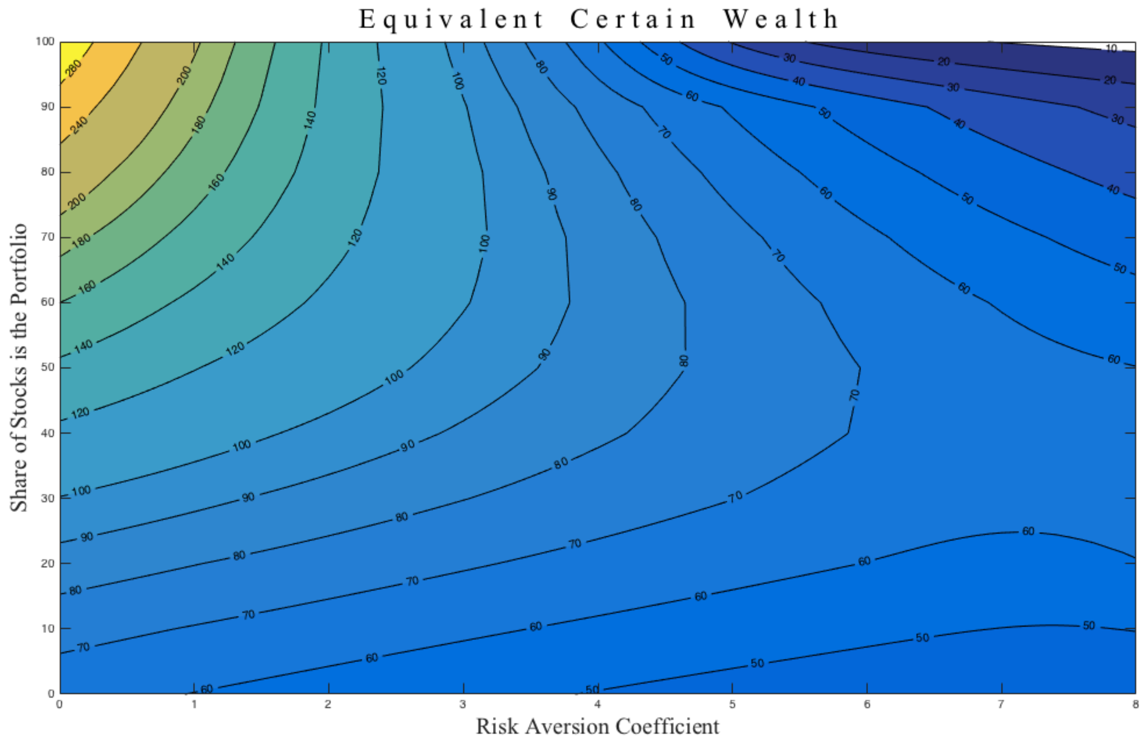
Table 6

Simulation with revised returns - Descriptive Statistics of Terminal Wealth and Equivalent Terminal Wealth

	Terminal Wealth Distribution					Equivalent Terminal Wealth depending on α			
Share of Equity	Mean	St Dev	Min	Median	Max	0	1	4	8
0	44,373	15,963	9,867	41,513	320,301	63,817	59,727	49,494	40,636
10	48,312	16,521	11,135	45,462	274,644	73,736	69,369	58,254	50,385
20	52,691	18,261	12,415	49,526	268,283	85,517	80,280	66,980	59,562
30	57,566	21,570	11,781	53,607	330,683	99,534	92,480	74,959	64,941
40	62,997	26,753	10,355	57,630	460,976	116,237	105,941	81,435	64,685
50	69,053	34,098	9,024	61,488	766,769	136,168	120,580	85,738	60,174
60	75,810	43,989	6,920	65,120	1,331,807	159,980	136,253	87,407	53,421
70	83,356	56,988	4,968	68,427	2,284,226	188,462	152,744	86,280	45,372
80	91,789	73,905	3,509	71,290	3,863,066	222,562	169,767	82,500	36,353
90	101,218	95,883	2,425	73,579	6,436,898	263,421	186,958	76,441	27,087
100	111,768	124,513	924	75,231	10,563,464	312,415	203,882	61,474	6,927
Glidepath	64,479	28,823	11,214	58,303	570,962	123,756	111,299	83,577	66,969
Inv Glidepath	73,986	43,839	4,502	63,461	1,273,559	149,996	127,615	79,720	43,815

(In thousands of dollars)

Figure A – Simulation (base case) - Equivalent Certain Wealth



This figure plots the Equivalent Certain Wealth (color/cohort) for a given CRRA level (x-axis) and Stock Allocation in the Portfolio (y-axis)

Figure B – Simulation (reduced case) - Equivalent Certain Wealth